

- 1 (a) Laplace Transformation - It is a technique for solving differential equations whereby the differential equation of time domain form is transformed to frequency domain.
- (b) Z Transformation - It is a technique of converting a discrete-time signal into a complex frequency domain representation.
- (c) Fourier Transformation - This is a technique that transforms a function of time to a frequency function.
- (d) Fourier series - It is a technique which is used to decompose signals into basis elements.

$$2(a) f(t) = 3t + 5 + \sin(t)$$

$$\begin{aligned} & \mathcal{L}\{3t + 5 + \sin(t)\} \\ &= \mathcal{L}\{3t\} + \mathcal{L}\{5\} + \mathcal{L}\{\sin(t)\} \\ &= \frac{3}{s^2} + \frac{5}{s} + \frac{1}{s^2+1} \end{aligned}$$

$$(b) f(t) = 14t^3 - e^{-t}$$

$$\begin{aligned} & \mathcal{L}\{14t^3\} - \mathcal{L}\{e^{-t}\} \\ &= 14\mathcal{L}\{t^3\} - \mathcal{L}\{e^{-t}\} \\ &= 14\left(\frac{3!}{s^4}\right) - \frac{1}{s+1} \\ &= \frac{84}{s^4} - \frac{1}{s+1} \end{aligned}$$

$$(c) f(t) = 3e^{-6t} \cdot \cos(2t)$$

$$\begin{aligned} &= 3\mathcal{L}\{e^{-6t} \cdot \cos(2t)\} \\ &= \frac{3(s+6)}{(s+6)^2+4} \\ &= \frac{3s+18}{(s+6)^2+4} \end{aligned}$$

$$(d) f(t) = 5\sin(3t)$$

$$\begin{aligned} &= 5\mathcal{L}\{\sin(3t)\} \\ &= \frac{5(3)}{s^2+9} \\ &= \frac{15}{s^2+9} \end{aligned}$$

$$(e) f(t) = 7e^{-3t} \sin(3t)$$

$$\begin{aligned} &= 7\mathcal{L}\{e^{-3t} \sin(3t)\} \\ &= \frac{21}{(s+3)^2+9} \end{aligned}$$

$$3(a)$$

$$F(s) = \frac{6}{s} + \frac{4}{s^2+4} + \frac{8}{s+4}$$

$$\begin{aligned} & \mathcal{L}^{-1}\left\{\frac{6}{s} + \frac{4}{s^2+4} + \frac{8}{s+4}\right\} \\ &= \mathcal{L}^{-1}\left\{\frac{6}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{4}{s^2+4}\right\} + \mathcal{L}^{-1}\left\{\frac{8}{s+4}\right\} \\ &= 6 + 2\sin(2t) + 8e^{-4t} \end{aligned}$$

$$(b)$$

$$F(s) = \frac{7}{(s+1)(s+3)}$$

$$\frac{7}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$$

$$\begin{aligned} 7 &= A(s+3) + B(s+1) \\ 7 &= A \cdot s(A+B) + 3A+B \end{aligned}$$

$$\begin{aligned} A+B &= 0 & A &= -B \\ 3A+B &= 7 \\ -3B+B &= 7 \\ -2B &= 7 & A &= \frac{7}{2} \\ B &= -\frac{7}{2} \end{aligned}$$

$$F(s) = \frac{7}{2(s+1)} - \frac{7}{2(s+3)}$$

$$\mathcal{L}^{-1}\left\{\frac{7}{2(s+1)} - \frac{7}{2(s+3)}\right\}$$

$$= \frac{7}{2}e^{-t} - \frac{7}{2}e^{-3t}$$

$$(c)$$

$$F(s) = \frac{1}{s^2+6s+5} = \frac{1}{(s+1)(s+5)}$$

$$\frac{1}{(s+1)(s+5)} = \frac{A}{s+1} + \frac{B}{s+5}$$

$$1 = A(s+5) + B(s+1)$$

$$1 = s(A+B) + 5A+B$$

$$A+B=0$$

$$A = -B$$

$$5A+B=1$$

$$-5B+B=1$$

$$-4B=1$$

$$B = -\frac{1}{4}$$

$$A = \frac{1}{4}$$

$$F(s) = \frac{1}{4(s+1)} - \frac{1}{4(s+5)}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{4(s+1)} - \frac{1}{4(s+5)} \right\}$$

$$= \frac{1}{4} e^{-t} - \frac{1}{4} e^{-5t}$$

$$(d) F(s) = \frac{s+3}{s(s+2)(s+4)}$$

$$\frac{s+3}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$s+3 = A(s+2)(s+4) + Bs(s+4) + Cs(s+2)$$

$$s+3 = As^2 + 6As + 8A + Bs^2 + 4sB + Cs^2 + 2Cs$$

$$s+3 = s^2(A+B+C) + s(6A+4B+2C) + 8A$$

$$A+B+C=0$$

$$6A+4B+2C=1$$

$$8A=3$$

$$A = \frac{3}{8}$$

$$B+C = -\frac{3}{8}$$

$$4B+2C = 1 - 6\left(\frac{3}{8}\right)$$

$$B = -\frac{1}{4}$$

$$C = -\frac{1}{8}$$

$$F(s) = \frac{3}{8s} - \frac{1}{4(s+2)} - \frac{1}{8(s+4)}$$

$$\mathcal{L}^{-1} \left\{ \frac{3}{8s} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{4(s+2)} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{8(s+4)} \right\}$$

$$= \frac{3}{8} - \frac{1}{4} e^{-2t} - \frac{1}{8} e^{-4t}$$

$$(e) F(s) = \frac{18}{s^2+81}$$

$$F(s) = \frac{9 \cdot 2}{s^2 + 9^2}$$

$$2 \mathcal{L}^{-1} \left\{ \frac{9}{s^2 + 9^2} \right\}$$

$$= 2 \sin(9t)$$

$$4(a) f(t) = 4t + 7 + \sin(4t)$$

$$F(z) = \frac{4z}{(z-1)^2} + \frac{7z}{z-1} + \frac{z \sin(4)}{z^2 - 2z \cos(4) + 1}$$

(b)

$$f(t) = 3 \cos(4t) + 2e^{-3t}$$

$$F(z) = \frac{3z(z - \cos(4))}{z^2 - 2z \cos(4) + 1} + \frac{2z}{z - e^{-3}}$$

$$(c) f(t) = 3e^{-6t} \cdot \cos(2t)$$

$$F(z) = \frac{3z}{z - e^{-6}} + \frac{z(z - \cos(2))}{z^2 - 2z \cos(2) + 1}$$

$$F(z) = \frac{3z(z - e^{-6} \cos(2))}{z^2 - 2z e^{-6} \cos(2) + e^{-12}}$$

5 (a)

$$F(s) = \frac{s+1}{s(s^2+6s+8)}$$

$$\frac{s+1}{s(s^2+6s+8)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$s+1 = A(s+2)(s+4) + Bs(s+4) + Cs(s+2)$$

$$s+1 = s^2(A+B+C) + s(6A+4B+2C) + 8A$$

$$A+B+C=0$$

$$6A+4B+2C=1$$

$$8A=1$$

$$A = \frac{1}{8}$$

$$B+C = -\frac{1}{8}$$

$$4B+2C = 1 - 6(\frac{1}{8})$$

$$B = \frac{1}{4}, C = -\frac{3}{8}$$

$$F(s) = \frac{1}{8s} + \frac{1}{4(s+2)} - \frac{3}{8(s+4)}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{8s} + \frac{1}{4(s+2)} - \frac{3}{8(s+4)} \right\}$$

$$f(t) = \frac{1}{8} + \frac{1}{4}e^{-2t} - \frac{3}{8}e^{-4t}$$

$$F(z) = \frac{z}{8(z-1)} + \frac{z}{4(z-e^{-2T})} - \frac{3}{8(z-e^{-4T})}$$

b) $F(s) = \frac{2(s+4)}{s(s+1)(s+2)}$

$$\frac{2(s+4)}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$2s+8 = A(s+1)(s+2) + Bs(s+2) + Cs(s+1)$$

$$2s+8 = s^2(A+B+C) + s(3A+2B+C) + 2A$$

$$A+B+C=0$$

$$3A+2B+C=2$$

$$2A=8 \quad A=4$$

$$B+C=-4$$

$$2B+C=2-3(4)$$

$$B=-6 \quad C=2$$

$$F(s) = \frac{4}{s} - \frac{6}{s+1} + \frac{2}{s+2}$$

$$\mathcal{L}^{-1} \left\{ \frac{4}{s} - \frac{6}{s+1} + \frac{2}{s+2} \right\}$$

$$f(t) = 4 - 6e^{-t} + 2e^{-2t}$$

$$F(z) = \frac{4z}{z-1} - \frac{6z}{z-e^{-T}} + \frac{2z}{z-e^{-2T}}$$

6 (a)

$$X(t) = 4e^{6t}u(t) + 2e^{-5t}\cos(6t)u(t)$$

$$F(\omega) = \frac{4}{-6+j\omega} + \frac{5+j\omega}{(5+j\omega)^2+36}$$

(b) $X(t) = 3e^{-2t}u(t) + 2e^{-5t}\sin(6t)u(t)$

$$F(\omega) = \frac{3}{2+j\omega} + \frac{6}{(2+j\omega)^2+36}$$

(c) $X(t) = f(t-6) + f(t+9)$

$$F(\omega) = F(\omega)e^{-j\omega 6} + F(\omega)e^{j\omega 9}$$

(d) $X(t) = 4e^{4t} - 3te^{-5t}u(t)$

$$F(\omega) = \frac{8}{16-\omega^2} - \frac{3}{(5+j\omega)^2}$$

$$7. (a) x(t) = \sum_{n=0}^5 a_n \cdot e^{jn5\pi t}$$

$$= a_0 + a_1 e^{j(1)5\pi t} + a_2 e^{j(2)5\pi t} + a_3 e^{j(3)5\pi t} + a_4 e^{j(4)5\pi t} + a_5 e^{j(5)5\pi t}$$

$$= 1 + 2e^{j5\pi t} + 4e^{j10\pi t} + 8e^{j15\pi t} + 10e^{j20\pi t} + 12e^{j25\pi t}$$

$$(b) x(t) = \sum_{n=-4}^4 a_n \cdot e^{jn5\pi t}$$

$$x(t) = 2 \sum_{n=0}^4 a_n e^{jn5\pi t}$$

$$= 2(1 + 2e^{j5\pi t} + 4e^{j10\pi t} + 8e^{j15\pi t} + 10e^{j20\pi t})$$

$$= 2 + 4e^{j5\pi t} + 8e^{j10\pi t} + 16e^{j15\pi t} + 20e^{j20\pi t}$$

$$(c) x(t) = a_0 + 3 \sum_{n=-5}^5 a_n \cdot \cos(3n\pi t)$$

$$= 1 + 3(2) (2 \cdot \cos(3\pi t) + 4 \cos(6\pi t) + 8 \cos(9\pi t) + 10 \cos(12\pi t) + 12 \cos(15\pi t) + 1)$$

$$= 7 + 12 \cos(3\pi t) + 24 \cos(6\pi t) + 48 \cos(9\pi t) + 60 \cos(12\pi t) + 72 \cos(15\pi t)$$

$$(d) x(t) = 4 \sum_{n=1}^4 b_n \cdot \sin(4n\pi t)$$

$$= 4(2 \sin(4\pi t) + 4 \sin(8\pi t) + 8 \sin(12\pi t) + 10 \sin(16\pi t))$$

$$= 8 \sin(4\pi t) + 16 \sin(8\pi t) + 32 \sin(12\pi t) + 40 \sin(16\pi t)$$

8.

Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{L}\right) + b_n \sin\left(\frac{n\pi t}{L}\right)$$

$$a_n = \frac{1}{L} \int_{-L}^L f(t) \cos\left(\frac{n\pi t}{L}\right) dt$$

$$b_n = \frac{1}{L} \int_{-L}^L f(t) \sin\left(\frac{n\pi t}{L}\right) dt$$

$$(a) \quad b_n = \frac{1}{L} \int_{-L}^L f(t) \sin\left(\frac{n\pi t}{L}\right) dt$$

$$= \frac{2}{L} \int_0^L f(t) \sin\left(\frac{n\pi t}{L}\right) dt$$

$$= \frac{2}{10} \int_0^{10} \cos(t) \sin\left(\frac{n\pi t}{10}\right) dt$$

$$(b) \quad a_0 = \frac{1}{L} \int_{-L}^L f(t) \cos\left(\frac{(0)\pi t}{L}\right) dt = 0$$

$$a_n = \frac{1}{L} \int_{-L}^L f(t) \cos\left(\frac{n\pi t}{L}\right) dt$$